Cautious label-wise ranking with constraint satisfaction

29èmes rencontres francophones sur la logique Floue et ses applications

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Our approach in a nutshell

What ?

Cautious label-ranking by rank-wise decomposition

How ?

☛ Rank-wise decomposition
☛ For each decomposition, predict a set of ranks using IP.
☛ Use Constraint Satisfaction (CSP) to :
  ☞ resolve inconsistencies
  ☞ remove impossible assignments

Why ?

☛ Recognizing hard instances to predict in order to avoid making mistakes $\Longrightarrow$ Making a cautious decision.
☛ few rank-wise approaches (except score-based) for this problem

Overview

- Introduction to Label ranking problem
- Our proposal in details
- Evaluation
  - Settings and Datasets
  - Experimental results
- Conclusions

Label ranking problem

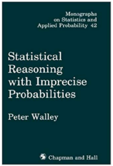
☞ Problem statement :

Let $\mathscr{K} = \{m_1, \ldots, m_K\}$ be a set of objects equipped with an order relation.

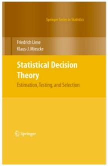
i.e. : ranked from “not relevant" → “relevant"

e.g. : Set of books ranked in order of preference (easy → hard)

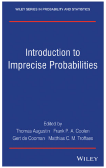
m_1



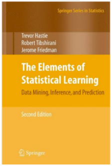
$\succ$ m_2



$\succ$ m_3



$\succ$ m_4



$\succ$ ...

A complete order relation on $\mathscr{K} \rightarrow \Lambda(\mathscr{K}) \subseteq \mathscr{K} \times \mathscr{K}$

$\Lambda(\mathscr{K})$: Set of complete rankings, $|\Lambda(\mathscr{K})| = K!$ (set of all permutations)

Label ranking problem

☞ The goal of label ranking problem :

Given a training data : $\mathscr{D} = \{\boldsymbol{x}_i, Y_i\}_{i=0}^{N} \subseteq \mathbb{R}^p \times \Lambda(\mathscr{K})$

Learning a complete ranking rule : $\varphi : \mathbb{R}^p \rightarrow \Lambda(\mathscr{K})$

☞ Example of training data :

X_1	X_2	Y
107.1	*Blue*	$m_1 \succ m_4 \succ m_2 \succ m_3$
−50	*Red*	$m_2 \succ m_3 \succ m_1 \succ m_4$
200	*Green*	$m_1 \succ m_3 \succ m_4 \succ m_2$
...	...	...

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Our proposal

Step ❶ Rank-wise decomposition

- for each label $m_i \in \mathscr{K}$, create a new data set $\mathbb{D}_{m_i}$

Step ❷ Cautious ordinal regression

- for each label $m_i \in \mathscr{K}$, predict a set of ranks using imprecise probabilities

Step ❸ Global inference with constraint satisfaction problem

- resolve inconsistencies
- remove impossibles assignments

Step ❶ : Rank-wise decomposition

$\mathbb{D}$

X_1	X_2	Y
107.1	*Blue*	$m_1 \succ m_3 \succ m_2 \succ m_4$
−50	*Red*	$m_2 \succ m_3 \succ m_1 \succ m_4$
200	*Green*	$m_1 \succ m_4 \succ m_3 \succ m_2$
…	…	…

$\mathbb{D}_1$

X_1	X_2	Y
107.1	*Blue*	1
−50	*Red*	3
200	*Green*	1
…	…	…

$\mathbb{D}_2$

X_1	X_2	Y
107.1	*Blue*	3
−50	*Red*	1
200	*Green*	4
…	…	…

$\mathbb{D}_3$

X_1	X_2	Y
107.1	*Blue*	2
−50	*Red*	2
200	*Green*	3
…	…	…

$\mathbb{D}_4$

X_1	X_2	Y
107.1	*Blue*	4
−50	*Red*	4
200	*Green*	2
…	…	…

FIGURE – Label-wise decomposition

☞ For each data set $\mathbb{D}_i$, solve an ordinal regression problem.

Step ❷ : Precise ordinal regression

$\mathbb{D}_1$

X_1	X_2	Y
107.1	*Blue*	1
−50	*Red*	3
…	…	…

$\mathbb{D}_2$

X_1	X_2	Y
107.1	*Blue*	3
−50	*Red*	1
…	…	…

$\mathbb{D}_3$

X_1	X_2	Y
107.1	*Blue*	2
−50	*Red*	2
…	…	…

$\mathbb{D}_4$

X_1	X_2	Y
107.1	*Blue*	4
−50	*Red*	4
…	…	…

Learning with an estimated probability $\widehat{\mathbb{P}}$

Given a training data : $\mathscr{D} = \{\boldsymbol{x}_i, y_i\}_{i=0}^{N} \subseteq \mathbb{R}^p \times \mathscr{K}$, where $\mathscr{K} = \{1,2,3,...,K\}$.

Risk minimizing [1, §2.4] :

$$\widehat{\varphi} := \arg\min_{\varphi(X)\in\Phi} \mathbb{E}_{\widehat{\mathbb{P}}}[\ell(Y,\varphi(X))] \qquad (1)$$

Ordinal regression

$$\widehat{\varphi} : \mathbb{R}^p \to \mathscr{K}$$

But, how can we do that with a set of probabilities $\widehat{\mathscr{P}}$?

$\mathbb{D}_1$

X_1	X_2	$\widehat{Y}$
107.1	*Blue*	{1,2}
−50	*Red*	{3}
…	…	…

$\mathbb{D}_2$

X_1	X_2	$\widehat{Y}$
107.1	*Blue*	{1,2,3}
−50	*Red*	{1}
…	…	…

$\mathbb{D}_3$

X_1	X_2	$\widehat{Y}$
107.1	*Blue*	{2}
−50	*Red*	{1,2}
…	…	…

$\mathbb{D}_4$

X_1	X_2	$\widehat{Y}$
107.1	*Blue*	{1,2,3,4}
−50	*Red*	{3,4}
…	…	…

Step ❷ : Cautious ordinal regression

$\mathbb{D}_1$

X_1	X_2	Y
107.1	*Blue*	1
−50	*Red*	3
…	…	…

$\mathbb{D}_2$

X_1	X_2	Y
107.1	*Blue*	3
−50	*Red*	1
…	…	…

$\mathbb{D}_3$

X_1	X_2	Y
107.1	*Blue*	2
−50	*Red*	2
…	…	…

$\mathbb{D}_4$

X_1	X_2	Y
107.1	*Blue*	4
−50	*Red*	4
…	…	…

Learning with a set of probabilities $\widehat{\mathscr{P}}$

Maximality criterion

$$m \succ m' \iff \inf_{P \in \mathscr{P}_{Y|x}} \sum_{y \in \mathscr{K}} P(y|x)(\ell_{m'}(y) - \ell_m(y)) \geq 0$$

Loss functions of choice $m \in \mathscr{K}$

$$\ell_m : \mathscr{K} \to \mathbb{R}$$

$\mathbb{D}_1$

X_1	X_2	$\hat{Y}$
107.1	*Blue*	{1, 2}
−50	*Red*	{3}
…	…	…

$\mathbb{D}_2$

X_1	X_2	$\hat{Y}$
107.1	*Blue*	{1, 2, 3}
−50	*Red*	{1}
…	…	…

$\mathbb{D}_3$

X_1	X_2	$\hat{Y}$
107.1	*Blue*	{2}
−50	*Red*	{1, 2}
…	…	…

$\mathbb{D}_4$

X_1	X_2	$\hat{Y}$
107.1	*Blue*	{1, 2, 3, 4}
−50	*Red*	{3, 4}
…	…	…

Step ❷ : Cautious ordinal regression

$\mathbb{D}_1$

X_1	X_2	Y
107.1	*Blue*	1
−50	*Red*	3
…	…	…

$\mathbb{D}_2$

X_1	X_2	Y
107.1	*Blue*	3
−50	*Red*	1
…	…	…

$\mathbb{D}_3$

X_1	X_2	Y
107.1	*Blue*	2
−50	*Red*	2
…	…	…

$\mathbb{D}_4$

X_1	X_2	Y
107.1	*Blue*	4
−50	*Red*	4
…	…	…

Learning with a set of probabilities $\widehat{\mathscr{P}}$

Maximality criterion

$$m \succ m' \iff \inf_{P \in \mathscr{P}_{Y|x}} \sum_{y \in \mathscr{K}} P(y|x)\left(\ell_{m'}(y) - \ell_m(y)\right) \geq 0$$

Loss functions of choice $m \in \mathscr{K}$

$$\ell_m : \mathscr{K} \to \mathbb{R}$$

But, what loss?

On the median in imprecise ordinal problems [3]

$\ell = L_1$ norm between ranks, loss of predicting rank j if k is true

$$\ell_j(k) = |j-k|$$

$\mathscr{P}$ described by lower/upper cumulative distributions $\underline{F}, \overline{F}$

$\mathbb{D}_1$

X_1	X_2	$\hat{Y}$
107.1	*Blue*	{1,2}
−50	*Red*	{3}
…	…	…

$\mathbb{D}_2$

X_1	X_2	$\hat{Y}$
107.1	*Blue*	{1,2,3}
−50	*Red*	{1}
…	…	…

$\mathbb{D}_3$

X_1	X_2	$\hat{Y}$
107.1	*Blue*	{2}
−50	*Red*	{1,2}
…	…	…

$\mathbb{D}_4$

X_1	X_2	$\hat{Y}$
107.1	*Blue*	{1,2,3,4}
−50	*Red*	{3,4}
…	…	…

Step ❷ : Cautious ordinal regression

$\mathbb{D}_1$

X_1	X_2	Y
107.1	*Blue*	1
−50	*Red*	3
…	…	…

$\mathbb{D}_2$

X_1	X_2	Y
107.1	*Blue*	3
−50	*Red*	1
…	…	…

$\mathbb{D}_3$

X_1	X_2	Y
107.1	*Blue*	2
−50	*Red*	2
…	…	…

$\mathbb{D}_4$

X_1	X_2	Y
107.1	*Blue*	4
−50	*Red*	4
…	…	…

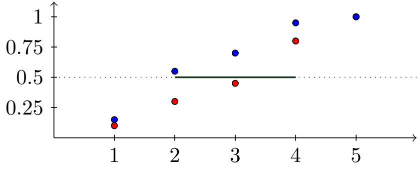
Why L_1 ?

- ☞ prediction is guaranteed to be an “interval" of ranks,
- ☞ it corresponds to the set of possible medians, i.e :

$$\widehat{R}_i = \left\{ j \in K : \underline{F}_{i(j-1)} \leq 0.5 \leq \overline{F}_{ij},\ \ \underline{F}_{i(0)} = 0 \right\}, \qquad (1)$$

An example of rank prediction

Rank j	1	2	3	4	5
$\overline{F}_j$	0.15	0.55	0.7	0.95	1
$\underline{F}_j$	0.1	0.3	0.45	0.8	1



Predicted rank for label: $\{2, 3, 4\}$

$\mathbb{D}_1$

X_1	X_2	$\widehat{Y}$
107.1	*Blue*	$\{1,2\}$
−50	*Red*	$\{3\}$
…	…	…

$\mathbb{D}_2$

X_1	X_2	$\widehat{Y}$
107.1	*Blue*	$\{1,2,3\}$
−50	*Red*	$\{1\}$
…	…	…

$\mathbb{D}_3$

X_1	X_2	$\widehat{Y}$
107.1	*Blue*	$\{2\}$
−50	*Red*	$\{1,2\}$
…	…	…

$\mathbb{D}_4$

X_1	X_2	$\widehat{Y}$
107.1	*Blue*	$\{1,2,3,4\}$
−50	*Red*	$\{3,4\}$
…	…	…

Step ❷ : Cautious ordinal regression

$\mathbb{D}_1$

X_1	X_2	Y
107.1	*Blue*	1
−50	*Red*	3
…	…	…

$\mathbb{D}_2$

X_1	X_2	Y
107.1	*Blue*	3
−50	*Red*	1
…	…	…

$\mathbb{D}_3$

X_1	X_2	Y
107.1	*Blue*	2
−50	*Red*	2
…	…	…

$\mathbb{D}_4$

X_1	X_2	Y
107.1	*Blue*	4
−50	*Red*	4
…	…	…

Cautious ordinal classifier [3]

$$\mathscr{R}_{m_i} : \mathbb{R}^p \to \widehat{Y} \subseteq \mathscr{K}$$

$\mathbb{D}_1$

X_1	X_2	$\widehat{Y}$
107.1	*Blue*	{1,2}
−50	*Red*	{3}
…	…	…

$\mathbb{D}_2$

X_1	X_2	$\widehat{Y}$
107.1	*Blue*	{1,2,3}
−50	*Red*	{1}
…	…	…

$\mathbb{D}_3$

X_1	X_2	$\widehat{Y}$
107.1	*Blue*	{2}
−50	*Red*	{1,2}
…	…	…

$\mathbb{D}_4$

X_1	X_2	$\widehat{Y}$
107.1	*Blue*	{1,2,3,4}
−50	*Red*	{3,4}
…	…	…

Step ❸ : Global inference with constraint satisfaction problem (CSP)

☞ Removal of impossible solutions

Consider the rank-wise $\mathscr{R}_i(\boldsymbol{x})$ cautious prediction of the first observation

$$\widehat{\mathscr{R}}_{m_1} = \{1,2\},\ \widehat{\mathscr{R}}_{m_2} = \{1,2,3\},\ \widehat{\mathscr{R}}_{m_3} = \{2\},\ \widehat{\mathscr{R}}_{m_4} = \{1,2,3,4\}$$

As $\mathscr{R}_{m_3}(\boldsymbol{x})$ has to take the single value $\{2\}$,then the others should not retain it :

$$\widehat{\mathscr{R}}^*_{m_1} = \{1\},\ \widehat{\mathscr{R}}^*_{m_2} = \{1,3\},\ \widehat{\mathscr{R}}^*_{m_3} = \{2\},\ \widehat{\mathscr{R}}^*_{m_4} = \{1,3,4\},$$

....until removing all inconsistencies, we get :

$$\widehat{\mathscr{R}}'_{m_1} = \{1\},\ \widehat{\mathscr{R}}'_{m_2} = \{3\},\ \widehat{\mathscr{R}}'_{m_3} = \{2\},\ \widehat{\mathscr{R}}'_{m_4} = \{4\}$$

✌ It is well-known as the *all different constraint* which forces every label-wise prediction to assume a value different from the value of every other.

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Datasets and experimental setting

Material and method

- 14 data sets issued from UCI repository [2].
- 10×10-fold cross-validation procedure.
- Binary decomposition + Naive imprecise classifier (NCC)
- Adaptive method to obtain the “optimal” value the hyper-parameter *s*
- Comparing with other precise approaches (LRT, RPC, SVM-LR)

Measuring results quality

Completeness (CP)

$$CP(\widehat{R}) = \frac{k^2 - \sum_{i=1}^{k} |\widehat{R}_i|}{k^2 - k}$$

- Max if one ranking possible
- Min if all rankings possible

Correctness (CR)

$$CR(\widehat{R}) = 1 - \frac{\sum_{i=1}^{k} \min_{\widehat{r}_i \in \widehat{R}_i} |\widehat{r}_i - r_i|}{0.5k^2}$$

- Equivalent to Spearman footrule if one ranking predicted

Completeness/Correctness trade-off

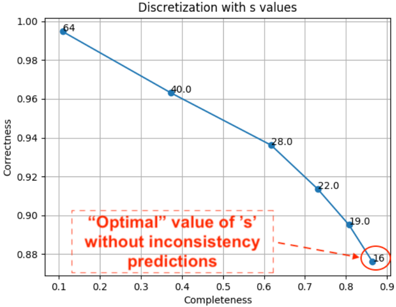

FIGURE – Evolution of the hyper-parameter s on glass

#	Data set	Type	#Inst	#Attributes	#Labels
a	authorship	classification	841	70	4
b	bodyfat	regression	252	7	7
c	calhousing	regression	20640	4	4
d	cpu-small	regression	8192	6	5
e	fried	regression	40768	9	5
f	glass	classification	214	9	6
g	housing	regression	506	6	6
h	iris	classification	150	4	3
i	pendigits	classification	10992	16	10
j	segment	classification	2310	18	7
k	stock	regression	950	5	5
l	vehicle	classification	846	18	4
m	vowel	classification	528	10	11
n	wine	classification	178	13	3

TABLE – Experimental data sets

Example : Inconsistency prediction :

$$\widehat{\mathscr{R}}'_{m_1} = \{1\},\ \widehat{\mathscr{R}}'_{m_2} = \{3\},\ \widehat{\mathscr{R}}'_{m_3} = \emptyset,\ \widehat{\mathscr{R}}'_{m_4} = \{4\}$$

Global solution is empty $\mathscr{R} = \emptyset$.

Experimental results

	LR-CSP-6	LRT	RPC	SVM-LR
a	93.90 ± 0.69 (1)	91.53 ± 0.31 (3)	93.21 ± 0.23 (2)	64.42 ± 0.36 (4)
b	54.12 ± 3.73 (1)	41.70 ± 1.48 (4)	50.43 ± 0.39 (3)	51.10 ± 0.49 (2)
c	61.05 ± 0.80 (1)	58.37 ± 0.28 (2)	51.85 ± 0.02 (3)	38.45 ± 0.02 (4)
d	68.72 ± 1.42 (1)	60.76 ± 0.30 (3)	61.93 ± 0.04 (2)	46.71 ± 0.87 (4)
e	99.20 ± 0.07 (2)	91.26 ± 0.06 (3)	99.92 ± 0.01 (1)	84.18 ± 2.67 (4)
f	91.95 ± 2.90 (1)	91.59 ± 0.47 (2)	90.83 ± 0.24 (3)	85.68 ± 0.33 (4)
g	79.21 ± 3.37 (2)	85.09 ± 0.46 (1)	74.86 ± 0.16 (3)	70.16 ± 0.46 (4)
h	99.36 ± 1.28 (1)	97.16 ± 0.55 (2)	92.75 ± 0.58 (3)	87.39 ± 0.37 (4)
i	91.31 ± 0.14 (3)	95.14 ± 0.05 (1)	94.12 ± 0.01 (2)	58.75 ± 2.71 (4)
j	91.20 ± 0.85 (3)	96.11 ± 0.10 (1)	94.52 ± 0.03 (2)	66.25 ± 3.05 (4)
k	88.63 ± 1.53 (2)	91.64 ± 0.27 (1)	82.23 ± 0.08 (3)	75.20 ± 0.17 (4)
l	85.29 ± 1.91 (3)	88.03 ± 0.44 (2)	89.24 ± 0.14 (1)	81.93 ± 1.00 (4)
m	88.23 ± 1.00 (1)	84.40 ± 0.62 (2)	72.88 ± 0.06 (3)	65.41 ± 1.21 (4)
n	98.20 ± 1.19 (1)	91.80 ± 0.87 (4)	94.58 ± 0.61 (2)	94.56 ± 0.50 (3)
avg.	85.03 ± 1.49(1.64)	83.18 ± 0.45(2.21)	81.67 ± 0.19(2.36)	69.30 ± 1.02(3.79)

TABLE – AVERAGE CORRECTNESS (%) COMPARED TO LR-CSP-6(%)

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Conclusions and Perspectives

- ✓ Our proposal is competitive w.r.t. other precise ones.
- ✓ We can use any imprecise classifier producing a set of probabilities.

- ✗ Deal with inconsistent predictions (~10% test dataset).
- ✗ Adapt the hyper-parameter s_i by imprecise classifier $\mathscr{R}_i$.
- ✗ Use other imprecise classifier (eg. continuous classifier)

References

Jerome FRIEDMAN, Trevor HASTIE et Robert TIBSHIRANI. *The elements of statistical learning*. Springer New York Inc., 2001.

A. FRANK et A. ASUNCION. *UCI Machine Learning Repository*. 2010. URL : http://archive.ics.uci.edu/ml.

Sébastien DESTERCKE. "On the median in imprecise ordinal problems". In : *Annals of Operations Research* 256.2 (2017), p. 375-392.


Thank
You!